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# The near-surface shear layer (NSSL) of the Sun: a theoretical model

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**Abstract.** We present a theoretical model of the near-surface shear layer (NSSL) of the Sun. Convection cells deeper down are affected by the Sun's rotation, but this is not the case in a layer just below the solar surface due to the smallness of the convection cells there. Based on this idea, we show that the thermal wind balance equation (the basic equation in the theory of the meridional circulation which holds inside the convection zone) can be solved to obtain the structure of the NSSL, matching observational data remarkably well.

**Keywords.** differential rotation, meridional circulation, convection, near-surface shear layer

## 1. Introduction

There is a layer just below the solar surface of thickness $\approx 0.05 R_\odot$ within which the angular velocity of the Sun decreases with radius by a few percent. This near-surface shear layer (NSSL) is clearly visible in any map of solar internal differential rotation produced by helioseismology: see Figure 3 of Choudhuri (2021b). There is not yet a consensus on what gives rise to this layer, although several different ideas have been proposed (Guerrero et al. 2013; Hotta et al. 2015; Matilsky et al. 2019). Here we summarize our work reported by Choudhuri (2021a) and Jha and Choudhuri (2021).

The theoretical ideas of how the two large-scale flow patterns of the Sun—the differential rotation and the meridional circulation—are produced are inter-related. A basic equation in the theory of the meridional circulation is the thermal wind balance equation, which holds within the body of the solar convection zone. It is often supposed that this equation breaks down near the solar surface surface, giving rise to a boundary layer in the form of NSSL. We propose the opposite view that this equation holds even near the surface and the change in the nature of convection due to the small scale height at the top layer causes the NSSL, of which the structure can be obtained by applying the thermal wind balance equation.

In §2 we summarize the basic theoretical ideas about the thermal wind balance equation and its implications for the changed nature of convection in the top layer within the solar convection zone. After presenting our results in §3, we end with our concluding remarks in §4.

## 2. Basic theoretical ideas

We first write down the thermal wind balance equation before explaining its significance. One standard form of this is

$$r \sin\theta \frac{\partial}{\partial z}\Omega^2 = \frac{g}{\gamma T}\left(\frac{\partial}{\partial \theta}\Delta T\right), \qquad (1)$$

where $z$ is the distance from the equatorial plane and the other symbols have their usual meanings, $\Delta T$ being the temperature variation with latitude to be discussed in more detail later.

Let us give an idea how the two sides of (1) arise. Since convective motions are influenced by the Coriolis force, the nature of convection changes slightly with latitude. The effect of the Coriolis force on radially moving fluid blobs being the least near the poles, the poles of the Sun are expected to be slightly hotter than the equatorial region. This would tend to drive a flow from a pole to the equator near the surface, i.e a clockwise meridional circulation in the northern hemisphere—which is opposite of what is seen. This effect is captured in the thermal wind term in the RHS of (1). On the other hand, if the centrifugal force varies along a straight line parallel to the rotation axis, that also can drive a meridional circulation which is anti-clockwise in northern hemisphere for solar-like angular velocity distribution. The centrifugal term in the LHS of (1) corresponds to this effect. The full dynamical equation of the meridional circulation has other terms (Choudhuri 2021b). However, a straightforward order-of-magnitude estimate suggests that the centrifugal term is much larger than these terms and can only be balanced by the thermal wind term, leading to (1).

Convective motions are influenced by the Coriolis force only if the convective turnover time is comparable to or more than the rotation period $\approx 27$ days. This is certainly not true for granules near the surface having turnover time of the order of a few minutes. We thus expect a layer at the top of the convection zone which is not affected by the Coriolis force. On the other hand, we believe that convective motions deeper down with much longer turnover times are affected by the Coriolis force. Although this is a gradual transition, we make the simplifying assumption that the Coriolis force is important when $r < r_c$ and negligible when $r > r_c$, where $r_c$ is a critical radius.

Now we write

$$T(r,\theta) = T(r,0) + \Delta T(r,\theta), \tag{2}$$

where $T(r,0)$ is the radial distribution of temperature as given by the standard solar model and $\Delta T(r,\theta)$ is the departure from it. Since convective motions in the top layer $r > r_c$ are not affected by rotation, we expect the temperature gradient there to be independent of latitude, i.e.

$$\frac{d}{dr}T(r,\theta) = \frac{d}{dr}T(r,0). \tag{3}$$

On substituting (2) in (3), we arrive at

$$\frac{d}{dr}\Delta T(r,\theta) = 0. \tag{4}$$

This suggests that $\Delta T(r,\theta)$ does not vary with $r$ in this top layer $r > r_c$ and we can write

$$\Delta T(r,\theta) = \Delta T(r_c,\theta). \tag{5}$$

In other words, the latitudinal variation of temperature at the critical radius $r_c$ gets mapped to the visible surface.

We now look at the RHS of (1). In the top layer $r > r_c$, the temperature $T$ drops sharply with increasing $r$ as we approach the surface, but (4) ensures that other quantities appearing in the RHS of (1) do not vary much with $r$ within this top layer. This implies that the RHS of (1) has to become very large close to the solar surface due to the drop in $T$. If (1) has to be satisfied, then the RHS of (1) also has to become very large just below the solar surface. For this to happen, $\Omega^2$ has to vary with $z$ rapidly. We suggest that this is what gives rise to the NSSL. This basic idea was suggested by Choudhuri (2021a) based on order-of-magnitude estimates. Detailed calculations were presented by Jha and Choudhuri (2021).

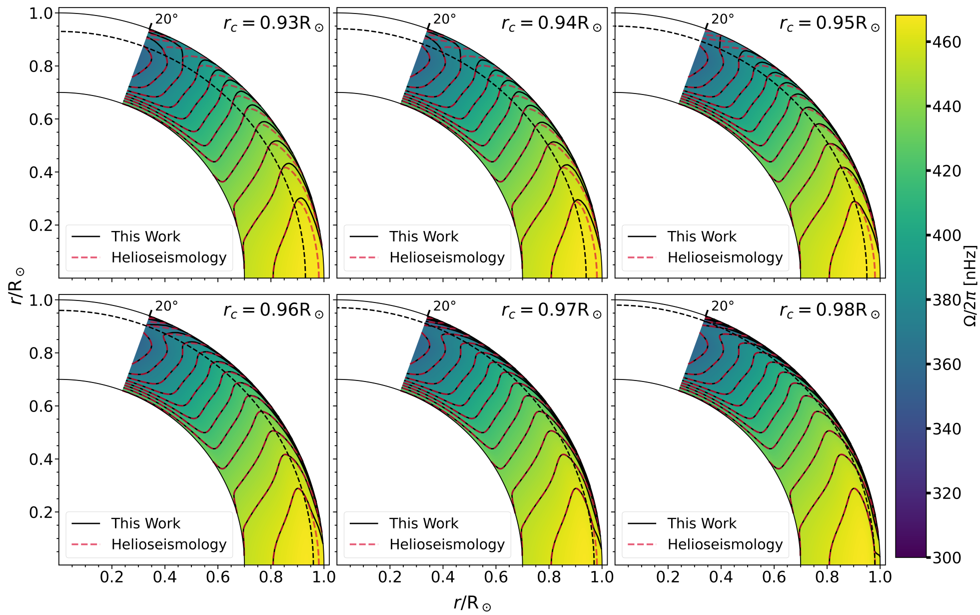


**Figure 1.** Contours of angular velocity Ω as given by helioseismolgy (dashed red curves) and by our model (solid black curves), for different assumed values of $r_c$. The different values of $r_c$ given in different panels are indicated by dashed black circles. From Jha and Choudhuri (2021).

## 3. Results

We calculate the structure of the NSSL in the top layer of the convection zone in the following manner. Assuming a particular value of $r_c$, we take Ω determined from helioseismology for $r < r_c$ and find $\Delta T$ in those deeper layers by using (1). This gives us $\Delta T(r_c, \theta)$ and we easily get $\Delta T(r, \theta)$ in the top layer $r > r_c$ from (5). Once we have $\Delta T(r, \theta)$ in the top layer $r > r_c$, we can use (1) to obtain Ω there, giving us the structure of NSSL. Figure 1 shows different contours of Ω we get by using different reasonable values of $r_c$. The solid black and dashed red lines show contours of Ω from our model and from helioseismology. They are the same below $r_c$ because we started our calculations in our model by using Ω from helioseismology below $r_c$. In the layer above $r_c$, we find that the two different sets of curves are very close to each other and our theoretical model gives the NSSL very nicely. We get best results when we assume $r_c = 0.96R_\odot$. Figure 2 shows the percentile error in our theoretical model in matching helioseismology data in the top layer for $r_c = 0.96R_\odot$. Except at the very surface, we find the error to be much less than 5%. This fantastic fit between theory and observations gives us confidence that we are on the right track.

A side result of our theoretical model is that we get a temperature variation at the solar surface. For the case $r_c = 0.96R_\odot$, the pole is found to be about 3 K hotter than the equator. It may be noted that Kitchatinov and Ruediger (1995) found a temperature difference of 5 K in their theoretical model of large-scale flows in the solar convection zone. There are some observational studies (Kuhn et al. 1988; Rast et al. 2008) suggesting that poles of the Sun are indeed slightly hotter than the equator. We hope that a more accurate observational study with modern techniques will be carried out in near future.

## 4. Conclusion

For the fist time, we are able to provide a theoretical model of the NSSL which matches observational data in quantitative detail. This model is based on the idea that the thermal wind

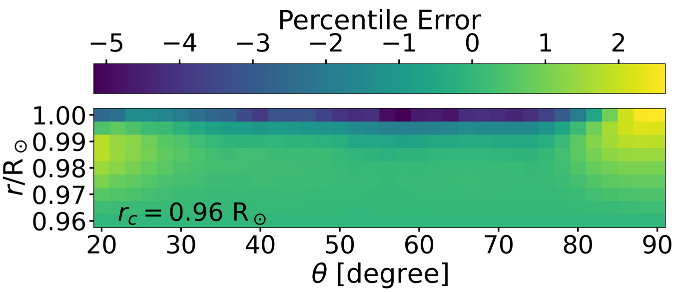


**Figure 2.** The percentile error of the theoretical model in matching the observational data of $\Omega$ for the case $r_c = 0.96 R_\odot$. From Jha and Choudhuri (2021).

balance equation holds even in the top layers of the convection zone just below the solar surface. We get the best results on assuming that $r_c = 0.96 R_\odot$ is the critical radius above which convective motions are not affected by the solar rotation, but below which they are affected. Perhaps it will be possible to check this through numerical simulations in future. One outcome of our model is that the poles of the Sun should be about 3 K hotter than the equator, another testable prediction.

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